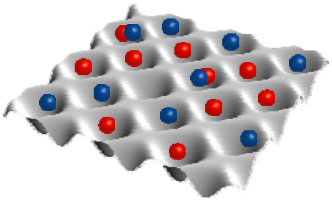
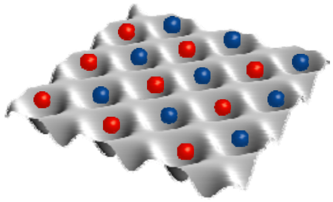
Superconductivity in the solid state

BCS Theory



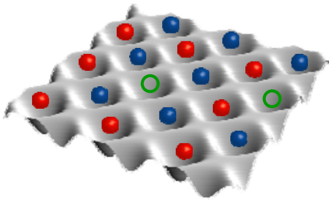
Attractive interactions via phonons
S- wave Cooper pairing

High T_C



Half filling
Anti-ferromagnet
isolator

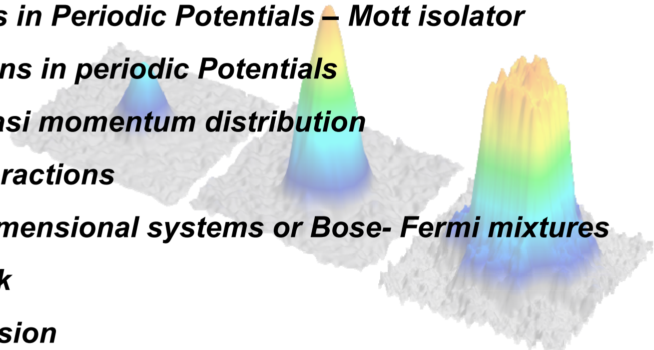
Remove some particles



Superconducting

Pairs have d-wave symmetry
Bonds resonate? Theoretical model?

Outline



- Periodic Potentials
- Bosons in Periodic Potentials – Mott isolator
- Fermions in periodic Potentials
  - Quasi momentum distribution
  - Interactions
- Low dimensional systems or Bose- Fermi mixtures
- Outlook
- Discussion

Optical Dipole Potentials

Energy of a dipole in an electric field:

$$U_{dip} = -\vec{d} \cdot \vec{E}$$

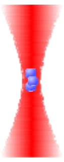
An electric field induces a dipole moment:

$$\vec{d} = \alpha \vec{E}$$

$$U_{dip} \propto -\alpha(\omega) I(\vec{r})$$

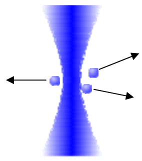
Red detuning:

Atoms are trapped in the intensity maxima



Blue detuning:

Atoms are repelled from the intensity maxima

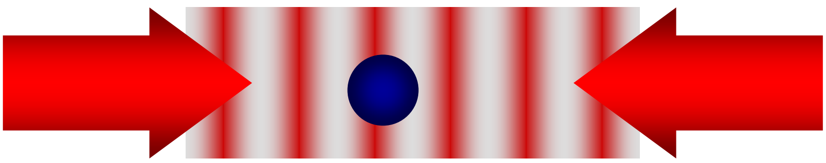


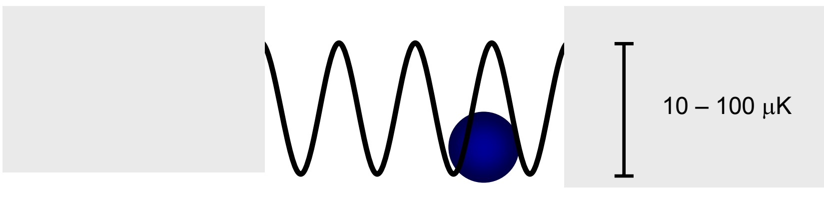
See R. Grimm et al., Adv. At. Mol. Opt. Phys. 42, 95-170 (2000).
Pioneering work by Steven Chu

Trapping Atoms in a Standing Wave

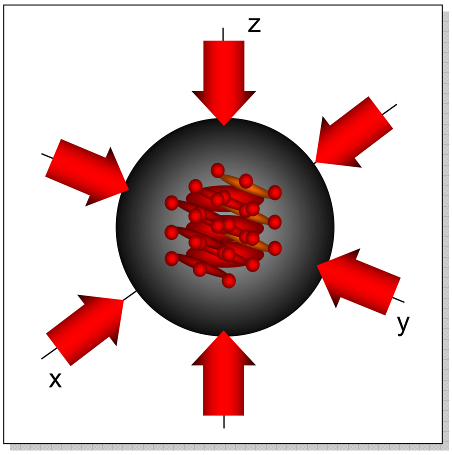
V. L. Letokhov, "Narrowing of the Doppler width in a standing light wave," JETP Lett. 7, 272 (1968).

Atoms are attracted to intensity maxima due to their polarizability.





3D Optical Lattice

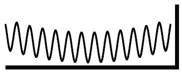


Atoms: ^{40}K (typically ~10^5)

Optical Lattice: 826nm

3D lattice with bosons: Munich/Mainz, NIST, ETH, Innsbruck, LENS, Hamburg

3D lattice with fermions: ETHZ, Hamburg, MIT, Mainz

•Potential = simple cubic lattice + confining potential = 

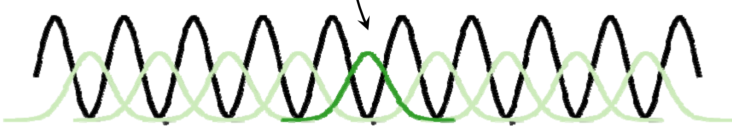
Wave Function in an Optical Lattice

Number of atoms on j^{th} lattice site

Phase of wave function on j^{th} lattice site

$$\Psi(x) = \sum_j A(x_j)\cdot w(x-x_j)\cdot e^{i\phi(x_j)}$$

Localized wave function on j^{th} lattice site



If there is a constant phase shift $\Delta\phi$ between lattice sites, the state is an eigenstate (Bloch wavefunction) of the lattice potential!

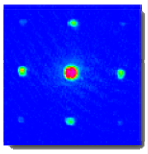
Quantum number characterizing these Bloch waves:

Crystal (Quasi-) momentum $q = \frac{2\hbar}{\lambda}\Delta\phi$

Time of flight interference pattern

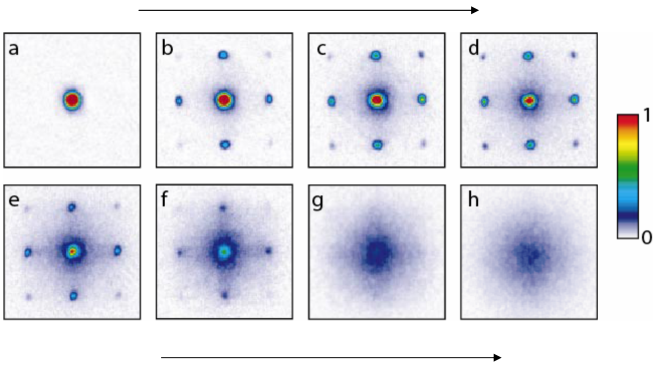
- Interference between all waves coherently emitted from each lattice site

a)



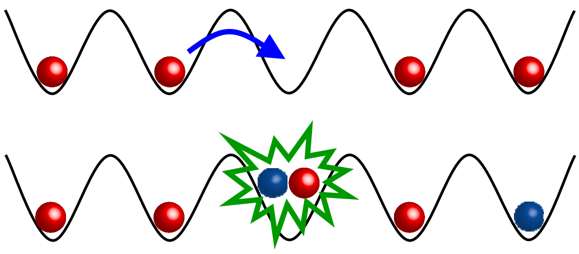
Mott isolator - a quantum phase transistion

Increasing lattice depth



M. Greiner, O. Mandel, T. Esslinger, T.W. Hänsch, M. Bloch, Nature 415, 39 (2002);

Processes in the lattice



tunneling

$-J\hat{a}_i^\dagger \hat{a}_{i-1}$

interaction

$+\frac{1}{2}U\hat{n}_i(\hat{n}_i-1)$

Hubbard model

$$\begin{aligned} H &= \int d^3x\, \hat{\psi}^\dagger(\mathbf{x}) \left(\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x})\right) \hat{\psi}(\mathbf{x}) \\ &+ \frac{1}{2}\frac{4\pi\hbar^2 a}{m} \int d^3x\, \hat{\psi}^\dagger(\mathbf{x})\hat{\psi}^\dagger(\mathbf{x})\hat{\psi}(\mathbf{x})\hat{\psi}(\mathbf{x}) \end{aligned}$$

Expanding the field operator in the Wannier basis of localized wave functions on each lattice site, yields :

$$\hat{\psi}(\boldsymbol{x}) = \sum_i \hat{a}_i w(\boldsymbol{x}-\boldsymbol{x}_i)$$

$$H = -J\sum_{\langle i,j\rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i-1) + \sum_i \varepsilon_i \hat{n}_i$$

Tunnelmatrix element/Hopping element

$$J = -\int d^3x\, w(\boldsymbol{x}-\boldsymbol{x}_i)\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{lat}(\boldsymbol{x})\right) w(\boldsymbol{x}-\boldsymbol{x}_j)$$

Onsite interaction matrix element

$$U = \frac{4\pi\hbar^2 a}{m}\int d^3x \left|w(\boldsymbol{x})\right|^4$$

D. Jaksch, C. Bruder, J.I. Cirac, C.W. Gardiner, P. Zoller , PRL 81, 3108 (1998);

Insulating vs. Superfluid state

$$H = -J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger \hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

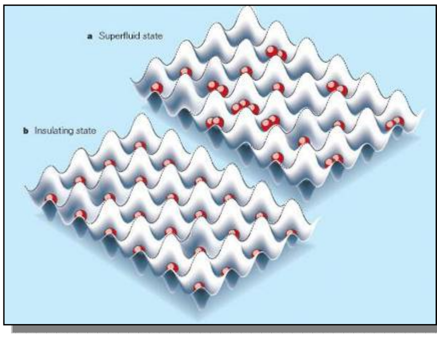
Atoms localised, Fock states on site
No phase coherence

$$H = -J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger \hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Atoms delocalised, site occupations possonian
Long range phase coherence

$$|\Psi_{Mott}\rangle \propto \prod_{i=1}^{M}\left(\hat{a}_i^\dagger\right)^n|0\rangle$$

Energy gap



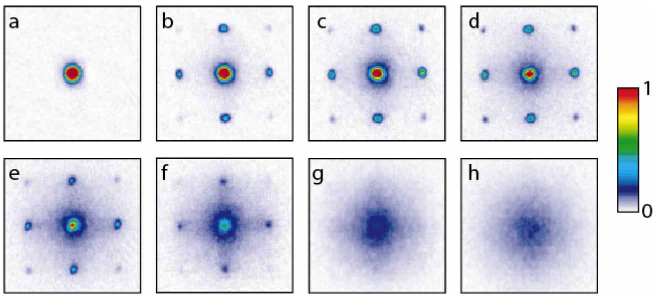
$$|\Psi_{SF}\rangle \propto \left(\sum_{i=1}^{M}\hat{a}_i^\dagger\right)^N|0\rangle$$

Gapless spectrum

Strongly correlated!

Momentum Distribution for Different Potential Depths

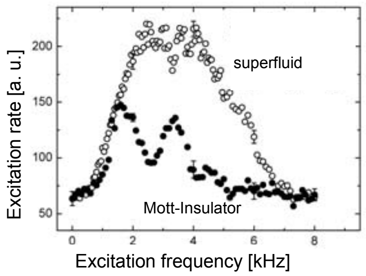
Superfluid, long phase coherence

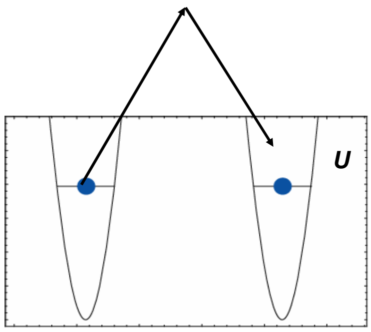


No coherence, Mott isolator?

M. Greiner, O. Mandel, T. Esslinger, T.W. Hänsch, M. Bloch, Nature 415, 39 (2002);

Mott-Insulator: Excitation Spectrum





T. Stöferle, H. Moritz, C. Schori, M. Köhl, T. Esslinger, Phys. Rev. Lett. 92, 130403 (2004);

Ideal Fermi gas in a 3D lattice

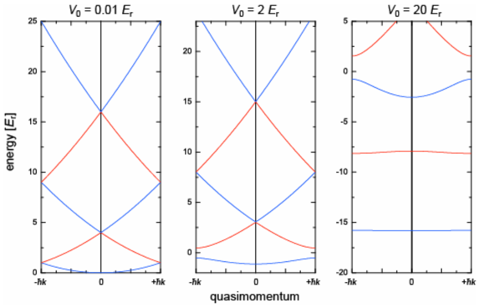
Noninteracting Fermions

Interactions

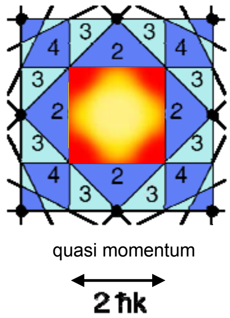
Low-Dimensional systems

Filling the Brillouin zone

Band structure



Brillouin zones
of a square lattice

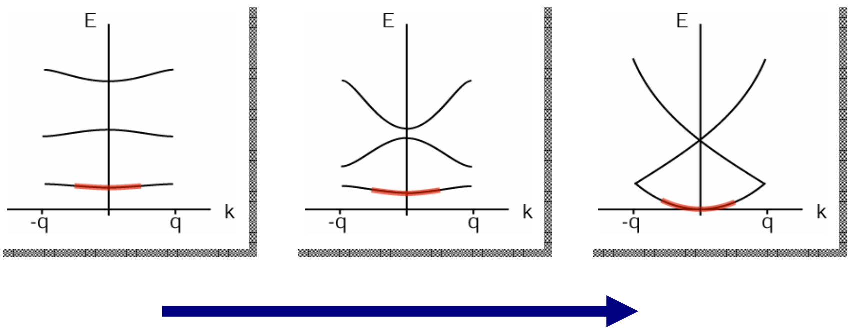


Adiabatic Expansion

Deep lattice

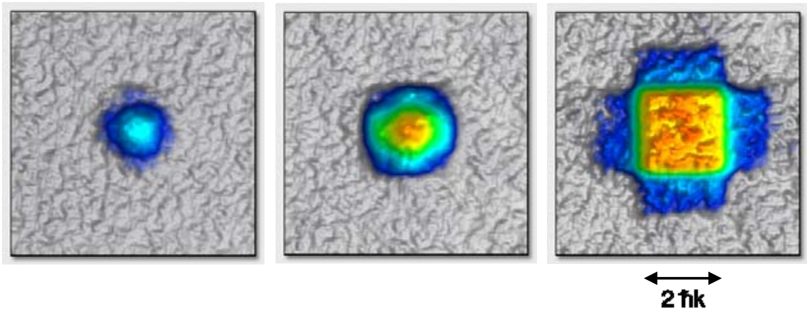
weak lattice

free atoms



- no transitions to higher bands
- quasi-momentum conserved (nearly)
- not adiabatic for many-body wavefunction

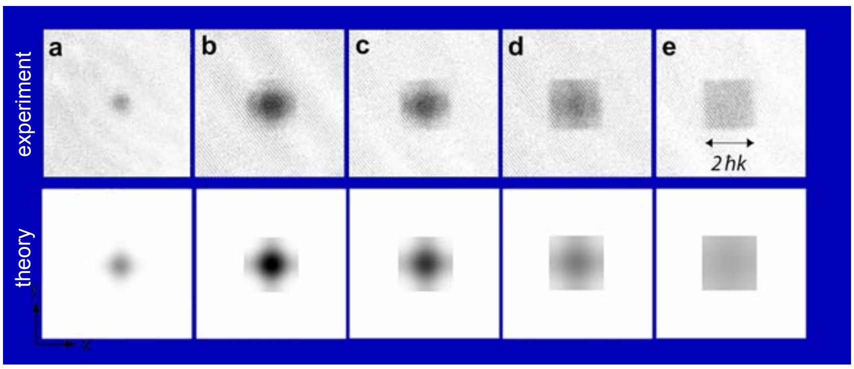
Fermi surfaces



"conductive state" filling "band insulator"

M. Köhl, H. M, T. Stöferle, K. Günter and T. Esslinger, PRL 94, 080403 (2005).

Observed Fermi surfaces



"conductive state" filling "band insulator"

M. Köhl, H. M, T. Stöferle, K. Günter and T. Esslinger, PRL 94, 080403 (2005).

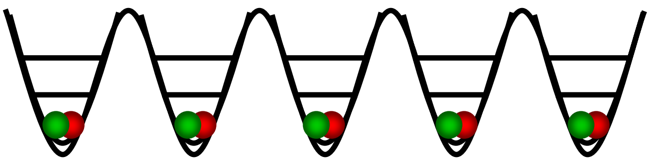
Ideal Fermi gas in a 3D lattice

Noninteracting Fermions

Interactions

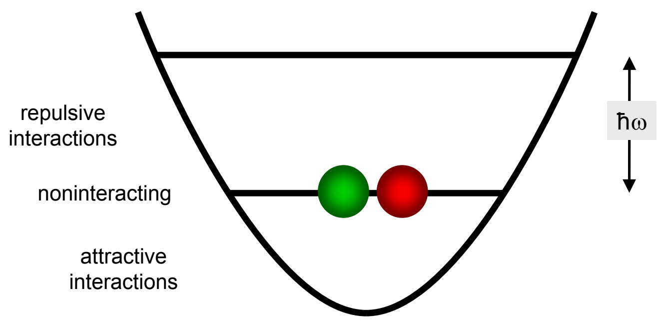
Low-Dimensional Systems

Interacting harmonic oscillator

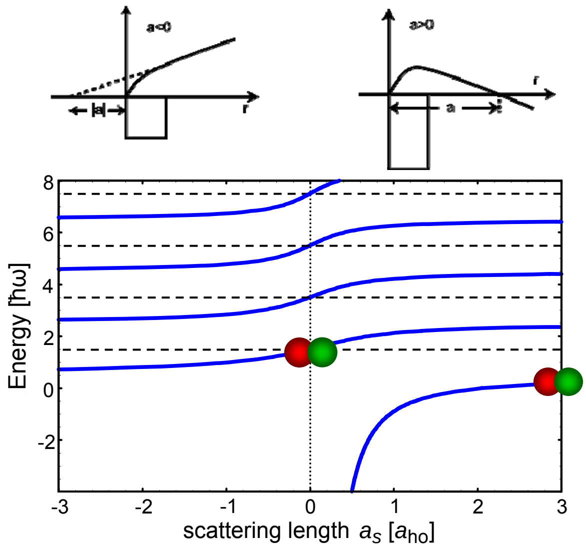


deep lattice = array of harmonic oscillators

Interacting harmonic oscillator

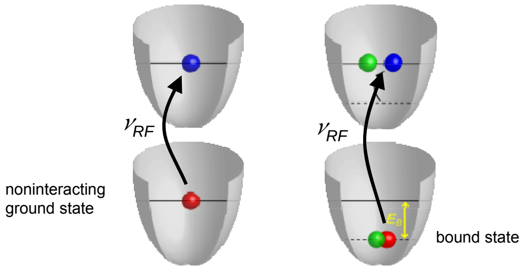


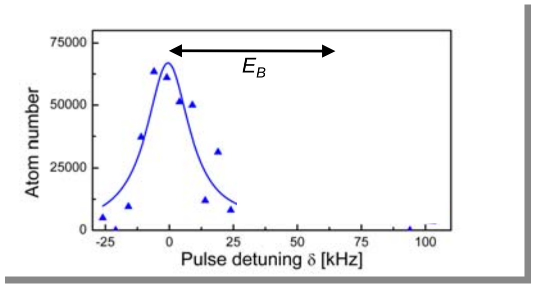
Creating molecules



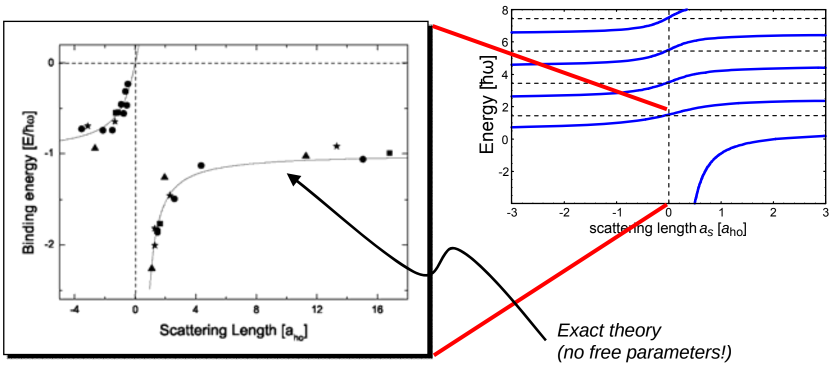
T. Busch et al., Found. Phys. 28, 549 (1998)

RF spectroscopy in the lattice





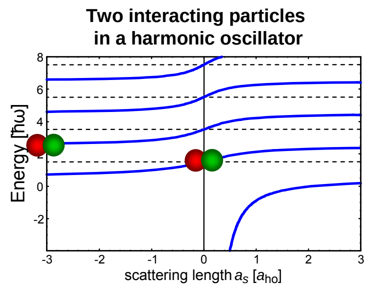
Measuring the binding energy

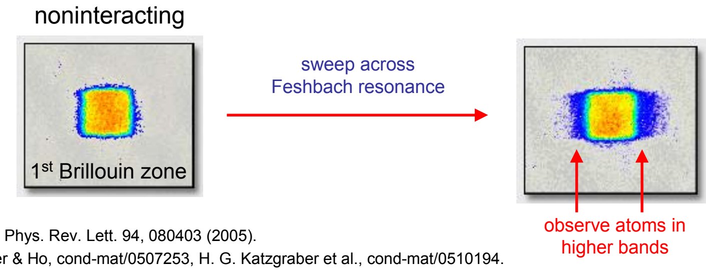


Fermionic atoms transform into bosonic molecules!

Thilo Stöferle, H. M., K. Günter, M. Köhl, T. Esslinger, Phys. Rev. Lett. 96, 040301 (2006)

Going the other direction …

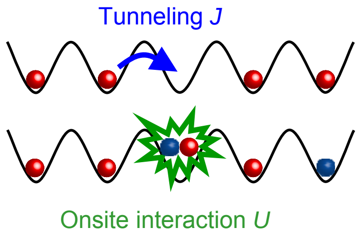




M. Köhl et al., Phys. Rev. Lett. 94, 080403 (2005).
Theory: Diener & Ho, cond-mat/0507253, H. G. Katzgraber et al., cond-mat/0510194.

Fermi-Hubbard model

$$H = -J \sum_{\substack{<i,j>,\sigma \\ \sigma=\uparrow,\downarrow}} \hat{c}^{\dagger}_{i,\sigma}\hat{c}_{j,\sigma} + \tfrac{1}{2}U\sum_{i}\hat{n}_{i\uparrow}\hat{n}_{i\downarrow} - \sum_{i,\sigma}(\mu-\varepsilon_{i,\sigma})\hat{n}_{i,\sigma}$$

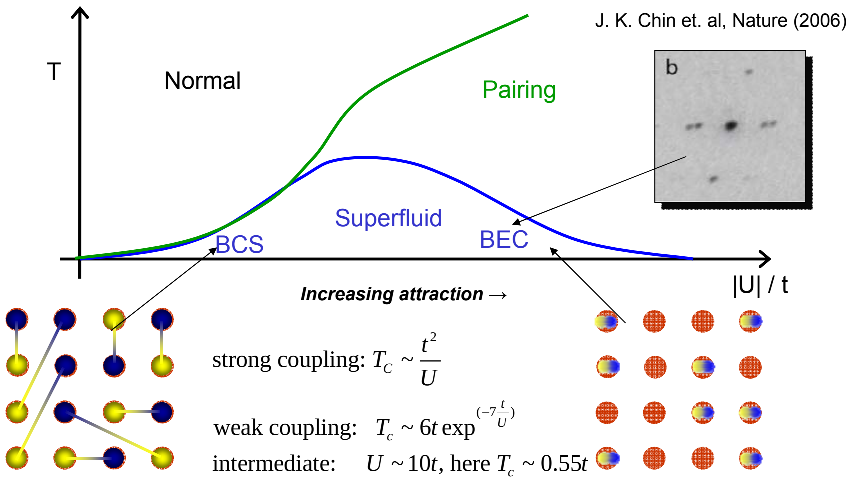


- interactions
- tunneling
- dimensionality
- filling
- confinement

- molecule formation

D. Jaksch, C. Bruder, J.I. Cirac, C.W. Gardiner, P. Zoller , PRL 81, 3108 (1998);

Attractive Phases



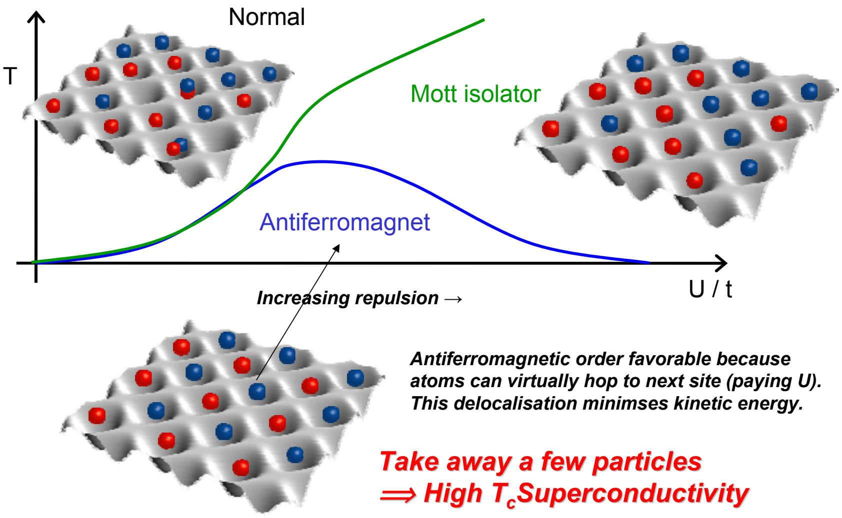
strong coupling: $T_C \sim \frac{t^2}{U}$

weak coupling: $T_c \sim 6t \exp^{(-7\frac{t}{U})}$

intermediate: $U \sim 10t$, here $T_c \sim 0.55t$

Repulsive Phases

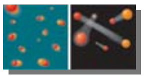
Half filling



Antiferromagnetic order favorable because atoms can virtually hop to next site (paying U). This delocalisation minimses kinetic energy.

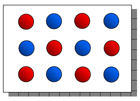
Take away a few particles
⟹ High T_c Superconductivity

Some obvious things to do



Superfluidity in the lattice, (also imbalanced)

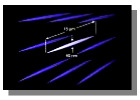
W. Hofstetter et al., PRL 89, 220407 (2002)



Mott insulating & antiferromagnetic phase

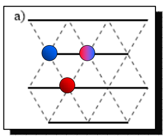
W. Hofstetter et al., PRL 89, 220407 (2002)
F. Werner et al., PRL 95, 056401 (2005)
E. Altman et al., PRA 70, 013603 (2004)

Some further directions with Fermions



Low-dimensional systems

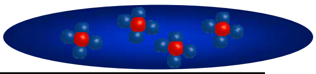
Exactly solvable BEC-BCS Crossover
Mapping from Bosons to Fermions and vice versa
Spin- Charge separation

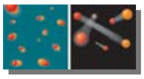


Trianglular lattices

Frustration
Spin liquids
Exotic Superconductivity

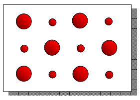
Bose-Fermi Mixtures





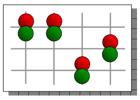
Boson mediated superfluidity

M. Cramer et al., PRL 93, 090406 (2004)
D.-W. Wang et al., PRA 72, 051604 (2005)



Supersolid phase

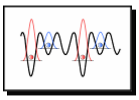
H.P. Büchler, G. Blatter, PRL 91, 130404 (2003)



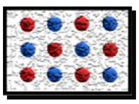
Dipolar molecules

Heteronuclear Feshbach molecules
Photoassociation

Novel Geometries



Superlattices



Disorder

with incommensurate superlattice
Speckle pattern